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Unipolar vs Bipolar Uncertainty

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Abstract. The article presents an application of fuzzy sets with triangular norms and balanced fuzzy sets with balanced norms to decision making modelling. We elaborate on a vector-based method for decision problem representation, where each element of a vector corresponds to an argument analysed by a decision maker. Vectors gather information that influence given decision making task. Decision is an outcome of aggregation of information gathered in such vectors. We have capitalized on an inherent ability of balanced norms to aggregate positive and negative premises of different intensity. We have contrasted properties of a bipolar model with a unipolar model based on triangular norms and fuzzy sets. Secondly, we have proposed several aggregation schemes that illustrate different real-life decision making situations. We have shown suitability of the proposed model to represent complex and biased decision making cases.
Keywords: decision making, balanced fuzzy sets, balanced norms, fuzzy sets, triangular norms

1 Introduction

Consumer decision making is an area extensively studied by specialists in various domains. In the pursuit of gaining the ability to model and predict human behavior neuroscientists unravel mysteries of human brain, social scientists study decisions in their social context, while economists relate decisions with money, markets and economies. The role of information science in research on decision making is critical, as it provides language for formal description and application of findings of the aforementioned domains. Information science faces a very challenging task, because quality of formal models could either hinder or enhance research in the other domains. Hence, it is of an utmost importance to work on frameworks for decision making modelling that are flexible and able to reflect real-world phenomena well.

It has been established that consumer decision making is a stimuli-driven process. Decision is an outcome of various influences recorded and evaluated consciously or unconsciously. Stimuli driving our actions are often called needs,

premises, factors, etc. We can generalize by saying that the decision making process is an outcome of information processing.

In this light, objective of the research underpinning this article is to elaborate on selected methods for describing and processing unipolar and bipolar information. We take under investigation fuzzy sets with triangular norms and balanced fuzzy sets with balanced norms. We apply a straightforward method for formal description of a decision problem, where forces influencing given decision are gathered in vectors. Secondly, we propose new information aggregation schemes for this model, which mimic consumer decision making processes.

The novelty aspect presented in this paper is the discussion on various information aggregation schemes for processing with balanced fuzzy sets and balanced norms. This analysis covers not only issues of information polarity, but also order of arguments' aggregation. Though the area of decision making itself is well-recognized, problems raised in this paper have not been analyzed in the context of consumer decision making.

The organization of this article is as follows. In Section 2 we introduce theoretical background of our decision making modelling framework. In particular, we elaborate on operators of interest. Section 3 presents application of standard and balanced norm to different decision making problems. A brief case study is provided to illustrate our approach. Section 4 concludes the paper and highlights future research directions.

2 Methodology

2.1 Brief literature review

Multiple criteria decision making is a thriving area of research with a wealth of contributions. Research relevant to the content of this study has been reported in [3], where a clear distinction between unipolar and bipolar information in a decision making problem has been addressed and elaborated on. A comprehensive perspective on fuzzy decision making could be found in [2], where the author summarizes key contemporary research streams of the domain. Interesting survey of multi-criteria decision-making can also be found in [6].

In a wealth of studies on multi-criteria decision making we can distinguish:

- unipolar,
- bipolar univariate,
- unipolar bivariate models.

Unipolar approach focuses on processing information of the same nature only, for example fuzzy sets, [13]. Bipolar univariate approaches, for example balanced fuzzy sets, [5], use single scale, typically divided by a neutral point to two zones to represent bipolar information. Unipolar bivariate models models, for example intuitionistic fuzzy sets, [1], use two separate scales, one for positive, second for negative information representation.

In this paper we look closely at the first two approaches. We are interested in modelling consumer decision cases, where attitudes could be expressed flexibly

and intuitively. The unipolar bivariate models are a viable alternative to the bipolar models. Both share the ability to describe information of different nature (positive and negative). However, in our opinion, the bipolar univariate model is more straightforward. Its advantage is that information is always related to the same scale.

2.2 Decision problem

Let us discuss certain decision problem. We have a decision maker, who is able to recognize all forces influencing give decision. Let us denote the number of such forces as n. Let us gather all these forces in an n-elements vector X:

$$X = [\ a_1,\ a_2,\ ...,\ a_n\ ]$$

The decision maker is able to evaluate polarity and intensity with which each force influences given decision. In this paper we discuss fuzzy sets and balanced fuzzy sets. Hence, scale is either $[0, 1]$ in the first case or $[-1, 1]$ in the second.

Causality is captured by introducing consecutive input arguments vectors. Following schema is modelled: input arguments are evaluated twice. Initially, they are gathered and processed in premises vector. Secondly, in priorities vector.

Premises describe attitudes towards certain features or possibilities associated with an object of the decision. In terms of cognitive perspective on decision making, premises are motivational stimuli, which elicit, control, and sustain certain behaviours. They are general factors relevant to the current motivational state. Premises can be somehow called an initial or an *a priori* motivation.

Priorities is the second term we use. This term is applied in the context of a second set of beliefs (second set of motivational stimuli evaluations). Priorities concern qualitatively exactly the same arguments as premises, but they are evaluated later, in the context of a particular product. Priorities allow to take into account reassessed attitudes towards a particular choice. Priorities provide a perspective of how one particular choice satisfies stated conditions. In this context, they may be perceived as an *a posteriori* motivation, arising when the decision maker reasons about a particular item. Of course, a set of priorities evaluations might be drastically different than premises.

Priorities are moderating premises. We are able to capture causality, because the decision takes into account not only final attitudes towards one particular product (priorities), but also general attitudes (gathered in premises vector). On such input data processing with chosen aggregators is performed.

Proposed technique mimics real consumer decision making processes, as reported by psychologists, for example in [10], where a decision could be viewed as an outcome of prior beliefs (here represented with premises) and current stimuli encouraging or discouraging us (priorities). Roots of mathematical representation of a decision making processes with two or more vectors representing causality could be found in [8].

2.3 Fuzzy connectives

Let us recall basic notions of fuzzy sets and generalization of fuzzy connectives min and max to triangular norms and conorms. We are expressing fuzzy sets in the form of membership functions. Namely, a fuzzy set A defined in the universe X is a mapping $\mu : X \to [0,1]$ or $\mu_{A,X} : X \to [0,1]$, if the names of the set and the universe should be explicitly stated. The Zadeh's model of fuzzy sets is clearly interpreted as $([0,1]^X, \max, \min, 1-)$, where $[0,1]^X$ states for mappings from the universe X into the unit interval $[0,1]$, i.e. $[0,1]^X$ is the space of membership functions, and max, min and $1-$ applied to membership functions implement qualitatively different operators on imprecise information.

Triangular norms and conorms Triangular norms have been introduced in [9] and then studied in [7, 11].

Triangular norms and conorms (t-norms and t-conorms in abbreviated form) are mappings $q : [0,1] \times [0,1] \to [0,1]$, where q stands for both t-norm t and t-conorm s, satisfying axioms of associativity, commutativity, monotonicity and boundary conditions. t-norms and t-conorms are dual operations in a sense that for any given t-norm t, we get the dual t-conorm s using the De Morgan's laws: $s(a,b) = 1 - t(1-a, 1-b)$ and $t(a,b) = 1 - s(1-a, 1-b)$

Let us focus on one well-known pair of dual t-norm and t-conorm: Lukasiewicz and bounded sum $(\max(0, x+y-1)/\min(x+y, 1))$ and on one pair of strict triangular norm and conorm generated by additive generators. We do not discuss decision making based on other standard triangular norms and conorms, because it is a topic thoroughly reported in the literature.

It is worth to recall that t-norms and t-conorms are bounded by min t-norm and max t-conorm, i.e. for any t-norm t, any t-conorm s and any $x, y \in [0,1]$ the following inequality holds:

$$t(x,y) \leq \min(x,y) \leq \max(x,y) \leq s(x,y) \tag{1}$$

Additive generators Triangular norms can be generated by additive or multiplicative generators, c.f. [7, 11]. Let us focus on additive generators for defining triangular norms and conorms. Let $f : [0,1] \to [0,d]$ be a non-decreasing mapping with $[0,d]$ being closed subintervals of the extended real semiline $[0,+\infty]$. Then the formula $f^{-1} : [0,+\infty] \to [0,1]$ such that $f^{-1}(y) = \sup\{x \in [0,1] : f(x) < y\}$ defines the pseudo-inverse of the mapping f. Similarly, if the mapping f is non-increasing, then the pseudo-inverse is defined by the formula $f^{-1} : [0,+\infty] \to [0,1]$ such that $f^{-1}(y) = \sup\{x \in [0,1] : f(x) > y\}$. We restrict our discussion to strictly monotonic and continuous bijections with $f(0) = 0$ for increasing mapping and $f(1) = 0$ for decreasing mapping f. Therefore we get $(f^{-1})^{-1} = f$ in the interval $[0,1]$ and $f^{-1}(y) = 1$ for $y > d$ for increasing mapping f and $f^{-1}(y) = 0$ for $y > d$ for decreasing mapping f. A mapping $q : [0,1]^2 \to [0,1]$ such that $q(x,y) = f^{-1}(f(x) + f(y))$ is a t-norm for decreasing f and t-conorm for increasing f. Moreover, such norms are monotonic and continuous, hence they are called *strict* norms as they are assuming strict monotonicity. Detailed discussion on additive generators is in [7].

We apply following additive generator of t-conorm and its pseudo-inverse mapping:

- $f : [0, 1] \to [0, +\infty]$ and $f^{-1} : [0, +\infty] \to [0, 1]$ such that:
  - $f(x) = \arcsin(x)$ and $f^{-1}(x) = \sin(\min(x, \pi/2))$.

2.4 Balanced connectives

Classical models of information processing were generalized by introducing new connectives defined on the unipolar unit interval and the unipolar unit square as well as the bipolar unit interval and the bipolar unit square. The former ones are uninorms and nullnorms with domain and codomain based on the unipolar unit interval, c.f. [7, 12]. The latter ones are balanced norms and conorms (b-norms and b-conorms) with domain and codomain based on the bipolar unit interval. Initial considerations on bipolar connectives were published in [4] and then balanced norms and conorms were introduced in [5]. Uninorms/nullnorms are fuzzy sets connectives, while b-norms/b-conorms are balanced fuzzy sets connectives.

Balanced fuzzy sets Fuzzy sets in represent both positive and negative information. Positive information is dispersed in the unit interval $(0, 1]$ while negative information is concentrated in the crisp point 0. The idea behind balanced fuzzy sets is to disperse negative information over the interval $[-1, 0]$. Such dispersion is analogical to the dispersion of positive information accumulated in the numeric value 1 into the (fuzzy) interval $[0, 1]$, c.f. [5].

Let us illustrate with a simple example what positive, negative and neutral information stands for. A customer needs to buy a car for transporting goods and for travelling with family. A lorry would provide superior cargo transportation (strong positive evaluation), but it would not be a good fit for travelling with the family (strong negative evaluation). In contrast, a notchback would be a good fit for travelling with the family (strong positive evaluation), but it would not be useful for goods transportation as its trunk space is not that impressive (strong negative evaluation). Finally, purchasing a minivan is a compromise as it meets moderately positive information in both aspects. On the other hand, a brand of the minivan may be unimportant, what would create a neutral information (neutral purchasing argument). Notice that strengths of arguments may differ and depend on circumstances, c.f. for instance [5].

Recalling basic notions, balanced fuzzy set is a system $\mathcal{F} = ([-1, 1]^X, \text{bmin}, \text{bmax}, \text{bneg}, I)$, where with X is a universe of discourse, bmin, bmax and bneg are balanced counterparts of min, max and $1-$ fuzzy connectives, c.f. Section 2.3:

- bmin equals min for positive arguments, equals max for negative arguments and vanishes for arguments of different signs,
- bmax equals to max for positive arguments, equals to min for negative arguments. bmax is in-between arguments of different signs and may be defined as equal to argument of greater absolute value. For arguments of opposite values it is undefined,
- bneg is the counterpart of $1-$ and may be defined as changing sign,

- I is a one place binverse operator. It does not have its fuzzy counterpart and may be defined as $I(a) = \text{sgn}(x) - x$ for $x \neq 0$ and undefined for $x = 0$.

In this paper connectives bmin and bmax are replaced with balanced norms and conorms, an inverse operator is not used.

Balanced norms and conorms Notice, that until the standard system $\mathcal{F} = ([-1,1]^X, \text{bnorm}, \text{bconorm}, -)$ is considered, the concepts of balanced norms and conorms (b-norms and b-conorms for short) are isomorphic with uninorms and nullnorms. Roughly speaking, a binverse operator I distinguishes systems $([0,1]^X, \text{uninorm}, \text{nullnorm}, \text{negation})$ and $([-1,1]^X, \text{b-norm}, \text{b-conorm}, \text{b-negation}, \text{b-inverse})$ and in this way its implementation may distinguish between b-norms/b-conorms and uninorms/nullnorms, c.f. [5, 12] for details. We use balanced norms and conorms rather than uninorms and nullnorms, because they model positive and negative information in a more intuitive way. Moreover, balanced norms and conorms represent positive and negative knowledge in a straightforward fashion, with the use of a single $[-1,1]$ scale, c.f. [4, 5].

In this paper we employ operations corresponding to triangular norms and conorms, which are generated by additive generators. A balanced norm T and balanced conorm S are mappings $Q : [-1,1] \times [-1,1] \to [-1,1]$, which are associative, commutative, increasing, and satisfy boundary conditions, c.f. [5]. Note that Q stands for both balanced norm and conorm. Boundary condition of balanced conorms is: $S(0,a) = a$ for all $a \in [-1,1]$. Due to associativity problem, the values $S(-1,1)$ and $S(1,-1)$ are both defined either as -1, or as 1, or as undefined, for instance c.f. [7]. For balanced norm T we have $T(1,a) = a$ whenever $a \in [-1,0]$ and $T(-1,a) = a$ whenever $a \in [-1,0]$. It is worth to underscore, that balanced norms must vanish for arguments of different signs. This is a direct conclusion drawn from monotonicity and boundary conditions. Note, that balanced norms and conorms restricted to the unipolar square domain $[0,1]$ are t-norms and conorms, respectively. Alike, balanced norms and conorms restricted to the unipolar negative square domain $[-1,0]$, when linearly transformed to the unit interval $[0,1]$, are t-conorms and t-norms, respectively.

Additive generators Alike in the case of triangular conorms, we can construct balanced triangular conorms using additive generators. Roughly speaking, an odd function being an additive generator of a symmetric balanced conorm (i.e. $S(a,b) = -S(-a,-b)$) is obtained by reflection of an additive generator of a triangular conorm in the origin of coordinate system, c.f. [5]. Additive generators of non-symmetric balance conorms are composed of different generators for nonpositive and nonnegative arguments. Namely, a function $F : [-1,1] \to [-d,d][-\infty,+\infty]$, which is strictly increasing continuous mapping with $[-d,d]$ being closed subintervals of the extended real line $[-\infty,+\infty]$, $F(-1) = -d$, $F(1) = +d$ and $F(0) = 0$. The function $S : [-1,1] \times [-1,1] \to ([-1,1] - \{(-1,1),(1,-1)\}$, $S(a,b) = F^{-1}(F(a) + F(b))$, is a balanced conorm, where F^{-1} is the pseudo-inverse of F (defined by analogy to Section 2.3), i.e. the function defined as follows: $F^{-1}(y) = \sup\{x \in [0,1] : F(x) < y\}$ for $y \geq 0$ and $F^{-1}(y) = \inf\{x \in [-1,0] : F(x) > y\}$ for $x < 0$.

In this study authors consider balanced conorm and norm generated by tangent additive generator:

- $F : [-1, 1] \to [-\infty, +\infty]$ and $F^{-1} : [-\infty, +\infty] \to [-1, 1]$ such that:
  - $F(x) = \tan(\pi/2 * x)$ and $F^{-1}(x) = 2/\pi * \arctan(x)$,

Additive generators of balanced norms are defined analogously to additive generators of balanced conorms. Reflection of an additive generator of a triangular norm in the origin of the coordinates' system gives an additive generator of a balanced symmetric norm. Gluing two different generators gives a generator of a non-symmetric balanced norm. It is worth to notice, that additive generators of balanced norms are undefined for 0. Additive generators of balanced norms are not discussed here. For the study presented in this paper De Morgan laws are sufficient for switching from balanced conorms to balanced norms.

3 Comparative overview of modelling capabilities of selected triangular and balanced norms and conorms

The goal of this section is to illustrate how the discussed theoretical framework could be employed for decision making modelling. To achieve clarity of presentation the discussion is limited to simple examples: fixed number of premises and priorities (either 5 or 10), repeated values of premises/priorities, computationally simple norms/conorms and derived operators, etc.

Such examples could be easily extended by adding more arguments or by analyzing various operators. A real-world source of information needed to conduct such study could be, for example, a survey.

3.1 Modelling decision making with triangular norms

The decision is obtained with a 2-step procedure. First, we aggregate premises with priorities. Second, we compute the final decision. Selected pairs of dual t-norms/t-conorms are involved in the process. Considering vectors of premises $[p_1, \ldots, p_5]$ and priorities $[r_1, \ldots, r_5]$, we come to the decision support (decision in the form of a number from the $[0, 1]$ interval):

$$d = s\Big(t(p_1, r_1), t(p_2, r_2), t(p_3, r_3), t(p_4, r_4), t(p_5, r_5)\Big) \tag{2}$$

where d is a decision support, t is a t-norm and s is the dual t-conorm. Here, associativity of the t-conorm s is assumed, which guarantees validity of this notation. Otherwise, since s is a two place operator, this formula should be rewritten, for instance, in the form $s(t(p_1, r_1), s(t(p_2, r_2), s(t(p_3, r_3), s(t(p_4, r_4), t(p_5, r_5)))))$.

Assuming that all premises are set to the value b and that k priorities change in the unit interval $x \in (0, 1]$ and other are equal to b:

$$p_1 = p_2 = p_3 = p_4 = p_5 = b, \quad r_1 = \ldots = r_k = x, \quad r_{k+1} = \ldots = r_5 = b \tag{3}$$

and utilize the formula 2 we get:

$$d = s\Big(t(b, x), t(b, \cdot), t(b, \cdot), t(b, \cdot), t(b, b)\Big) \tag{4}$$

where $\cdot$ stands either for x, or for b, depending on the value k.

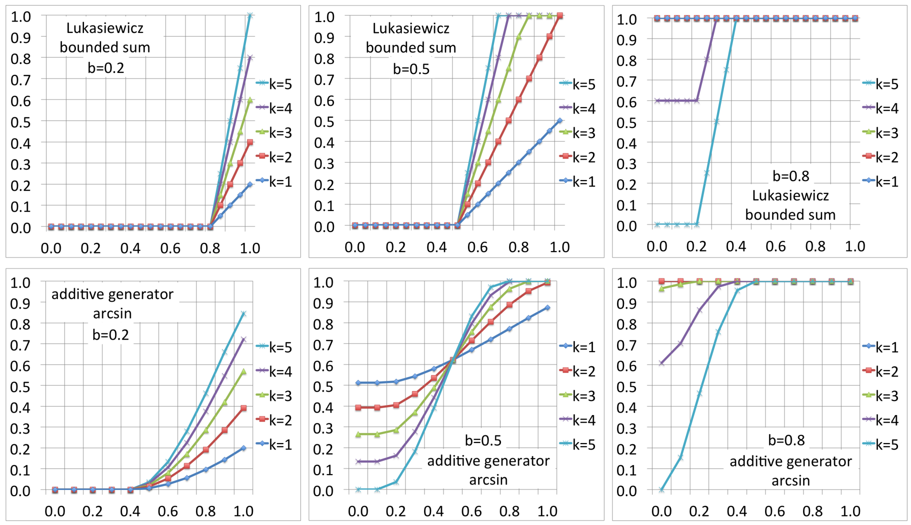

Fig. 1. Modelling decision making with Lukasiewicz/bounded sum (top row) and arcsine generated dual norms (bottom row). Decision is based on 5 pairs of premise/priority. All premises are equal b. Priorities are vectors with k arguments of strength x, remaining $5-k$ elements are equal b.

Standard triangular norms Let us start with presentation of decision making results with selected pair of dual t-norms/t-conorms: Lukasiewicz/ bounded sum. We consider vectors of premises and priorities and utilize the scheme for computing decision support as in Formulas 2 and 3.

Figure 1 demonstrates how a modification of arguments of one type influences the decision assuming other type arguments on a fixed level for a variety of operators. The following observations regarding plots in Figure 1 can be made: Lukasiewicz t-norm/bounded sum t-conorm are insensitive for values of arguments low (t-norms) or high (t-conorms) enough. These operators achieve lower/upper saturation state for values of arguments smaller/greater than some threshold. The impact of upper and lower thresholds is distinctly visible in differences between plots, where $b = 0.2$ and $b = 0.8$. The first one has all premises equal to 0.2. As a result, only for very strong priorities (greater than 0.8) we can get the decision greater than 0. In contrast, for strong arguments (like $b = 0.8$) the decision gets drawn to the upper threshold level and it is equal to 1, unless premises are combined with very small priorities. It happens only for $k = 5$ and $k = 4$, which are in this decision problem relatively big numbers of weak priorities. The middle plot (for $b = 0.5$) shows decisions for moderate arguments and transition between the extremes. In this case both lower and upper thresholds are clearly visible. Right plot concerns strong positive premises ($b = 0.8$). For such strong premises set the only case where the consumer is not able to make a positive decision is when all priorities are very small (for $k = 5$ and x in the range $[0, 0.2]$).

Second pair of operators of interest are arcsine generated dual norms, for which decision making results are in the second row of Figure 1. We see similarity between plots in the first and the second row. For both pairs of operators decisions vanish (are set to 0) for small input arguments and get saturated (to 1) for high arguments. In the case of Lukasiewicz t-norm and bounded sum the critical (threshold) point is sharp and easy to identify on the plot, while for arcsine generated dual norms the transition between 0 and 1 is smooth.

First column in Figure 1 shows that if positive premises influencing the decision are of weak intensity (first column, $b = 0.2$), then the consumer makes a positive decision only if all priorities are close to 1. This agrees with a common sense. We expect weak positive premises to vote over priorities. In contrast, if premises were higher at the beginning (middle and right plots, $b = 0.5$ and $b = 0.8$ respectively) it is easier to reach positive decision. Also in real life, we would expect that if our convictions about certain issue were high at start, we would have necessary motivational background to encourage certain decisions. In our model, priorities could be interpreted as the hot system described by W. Mischel in [10]. Evaluation of priorities under biased or emotional circumstances could lead to unexpected strong feelings towards certain products, for example chocolates. Though the rational cold system, here represented with premises, knows that in order to be healthy we should not be eating chocolate, strong positive stimuli of the hot system (priorities) could outvote it.

3.2 Discussion on selected balanced norms

The application of balanced norms and conorms is an objective of this paper and it is an original input of the authors to the field of decision making based on both positive and negative forces. We use balanced norm and conorm associated with $\tan(\pi/2\ \cdot)$ in the $[-1, 1]$ domain.

We consider vectors of premises $[p_1, \ldots, p_{10}]$ and priorities $[r_1, \ldots, r_{10}]$. The elements of both vectors are assumed to hold positive and negative values. It is assumed that the corresponding elements in vectors of premises and priorities are either both positive or both negative. Any case of different types of values (negative and positive) in corresponding elements is not considered.

Alike in Section 3.1, there are 5 pairs of positive premises/priorities. Positive values of premises are constant and equal b. k priorities change in the unit interval $x \in [0, 1]$ - the reference is on the OX axis. $5 - k$ positive values of priorities remain fixed at the same level as premises (b). There are j pairs of negative premises/priorities. Pairs of negative premises and priorities are set to the same negative constant value c, while the other $5 - j$ pairs of premises and priorities are not considered.

Unlike in Section 3.1, computing decision support may not be straightforward. Balanced norms obtained with generators such as arctangent hyperbolic and tangent are associative and commutative with the only exception for numeric values of different signs, c.f. Section 2.4. However, associativity and commutativity are not satisfied by norms generated, for instance, by arcsine. Therefore,

computing with balanced norms following Formulas 2 and 3 is not valid. Instead we employ more complex assumptions.

Let us remind that at first corresponding premises and priorities are moderated with a balanced norm. Secondly, results of such balanced norm aggregation are aggregated with a balanced conorm. Let us introduce two different schemes of aggregation with a balanced conorm are applied: batch aggregation and one-by-one aggregation.

Batch aggregation - positive and negative results moderated with a balanced norm are aggregated separately with a balanced conorm. Then, both results computed by the balanced conorm are aggregated with the same conorm:

$$S\Big(S\Big(T(p_{i_1}, r_{i_1}), \ldots, T(p_{i_5}, r_{i_5})\Big), S\Big(T(p_{i_6}, r_{i_6}), \ldots, T(p_{i_{5+j}}, r_{i_{5+j}})\Big)\Big) \tag{5}$$

where:
- $p_{i_1}, r_{i_1}, \ldots, p_{i_5}, r_{i_5} \geqslant 0$ and $p_{i_6}, r_{i_6}, \ldots, p_{i_{5+j}}, r_{i_{5+j}} \leqslant 0$
- T is a balanced norm and S is a balanced conorm.

This formula is valid due to associativity of balanced triangular norms for non-negative arguments and for non-positive arguments as well as commutativity of balanced conorm for two arguments.

One-by-one aggregation - consecutive results of a moderation with balanced norm are aggregated with a dual balanced conorm according to the following formula:

$$S\Big(S\Big(\ldots\Big(S\Big(T(p_1, r_1), T(p_2, r_2)\Big), T(p_3, r_3)\Big), \ldots\Big), T(p_{5+j}, r_{5+j})\Big) \tag{6}$$

Following orderings of negative and positive values and corresponding vectors of premises and priorities are proposed:

1. 5 positive values first and then 5 negative ones:
   $p_1 = p_2 = p_3 = p_4 = p_5 = b$ and $p_6 = p_7 = p_8 = p_9 = p_{10} = c$
   $r_1 = r_2 = r_3 = r_4 = r_5 = x$ and $r_6 = r_7 = r_8 = r_9 =_{10} = c$
   $Prem = [b, b, b, b, b, c, c, c, c, c]$ and $Prior = [x, x, x, x, x, c, c, c, c, c]$
2. 5 negative values first and then 5 positive ones:
   $p_1 = p_2 = p_3 = p_4 = p_5 = c$ and $p_6 = p_7 = p_8 = p_9 = p_{10} = b$
   $r_1 = r_2 = r_3 = r_4 = r_5 = c$ and $r_6 = r_7 = r_8 = r_9 = r_{10} = x$
   $Prem = [c, c, c, c, c, b, b, b, b, b]$ and $Prior = [c, c, c, c, c, x, x, x, x, x]$
3. 5 positive values first and then j negative ones:
   $p_1 = \ldots = p_5 = b$ and $p_6 = \ldots = p_{5+j} = c$
   $r_1 = \ldots = r_k = x, \quad r_{k+1} = \ldots = r_5 = b$ and $r_6 = \ldots = r_{5+j} = c$
   $Prem = [b, b, b, b, b, c, c, \ldots]$ and $Prior = [x, \ldots, x, b, \ldots, b, c, c, \ldots]$
4. alternate positive/negative values beginning with a positive one:
   $p_1 = p_3 = p_5 = p_7 = p_9 = b$ and $p_2 = p_4 = p_6 = p_8 = p_{10} = c$
   $r_1 = r_3 = r_5 = r_7 = r_9 = x$ and $r_2 = r_4 = r_6 = r_8 = r_{10} = c$
   $Prem = [b, c, b, c, b, c, b, c, b, c]$ and $Prior = [x, c, x, c, x, c, x, c, x, c]$
5. alternate positive/negative values beginning with a negative one:
   $p_1 = p_3 = p_5 = p_7 = p_9 = c$ and $p_2 = p_4 = p_6 = p_8 = p_{10} = b$
   $r_1 = r_3 = r_5 = r_7 = r_9 = c$ and $r_2 = r_4 = r_6 = r_8 = r_{10} = x$
   $Prem = [c, b, c, b, c, b, c, b, c, b]$ and $Prior = [c, x, c, x, c, x, c, x, c, x]$

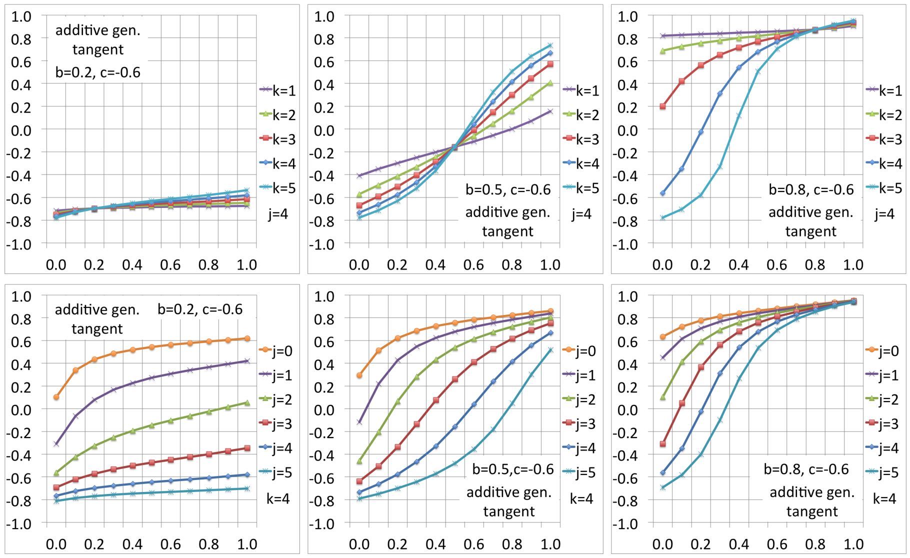

Fig. 2. Decision making modelling with dual balanced norms associated with tangent. Batch aggregation with scheme no. 3 is applied. Decision problems are based on 5 pairs of positive premises of strength b and priorities of strength x (on the OX axis) and j negative pairs of premise/priority both of strength c.

Various aggregation scenarios are constructed to represent different real-world decision making problems. We aimed at proposing processing schemes that would have the capacity to illustrate a wealth of real-life situations.

Figure 2 illustrates decision making results based on balanced norms associated with tangent additive generator. Tangent-based norms overestimate weak arguments. Emphasizing effect is clearly visible in two top lines (for zero or few negative input pairs: $j = 0$, $j = 1$ and $j = 2$) in the bottom row. Emphasizing effect causes also that in the top left plot (for $b = 0.2$ and $c = -0.6$) decisions computed with tangent are higher, than if we would have had applied arcsine or arctangent hyperbolic-based connectives. Tangent-based operators reflect well consumer behaviour. Overestimation of weak arguments is a behavioural bias of decision making - persistent deviation we observe. If we aggregate strong premises/priorities the strengthening effect is not present and results are in-between decisions computed with arcsine and arctangent hyperbolic norms.

With growth of the number of negative arguments decisions become weaker. Lines on the left plot in the bottom row of Figure 2 are placed one above another. For $j = 0$ lines are higher than for $j = 1$, $j = 2$, $j = 3$, $j = 4$ and $j = 5$ and so on. Decisions computed with tangent are relatively high, but with an upper boundary. Saturation effect is well-visible in right plots, which present decision cases with strong positive result. Similarly, we see a clear threshold for a negative decision in top left plot (for $j = 4$, $b = 0.2$ and $c = -0.6$).

Other norms are not considered here due to space constraints.

4 Conclusion

The article presents an approach to decision making modelling. The method is based on aggregation of premises influencing given decision. We emphasize the distinction between the two schemes: unipolar and bipolar. The unipolar approach, which could be represented with fuzzy sets and triangular norms, is able to process positive information only. The bipolar approach realized with balanced fuzzy sets and balanced norms overcomes this limitation and could be applied to represent positive and negative premises of different intensity. Building on the assumption that there is a relation between decision and an order, in which consumer acknowledges premises influencing this decision we have discussed various aggregation schemes that mimic different real-life decision making scenarios. Presented ideas have been illustrated with a brief case study, where we apply and compare different operators in several decision making problems. The study confirms advantageous modelling capability of the proposed methods. In future, we plan to continue the development of this framework. In particular, we plan to use it to describe basic behavioral biases in decision making.

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